# Empirical Analysis of the Robustness of the Size Effect

## Summary

For many years academics and practitioners in the field of financial economics have strived to identify the most significant factors which influence the pricing and consequently the returns of a financial asset. In this context the Capital Asset-Pricing Model (CAPM) by Sharpe (1964), Lintner (1965) and Black (1972) has long enjoyed great popularity among practitioners and academics. The central conclusion of this model is that a firm's expected equity return - and therefore its price - should relate linearly to its market beta, which can be interpreted as a measure of the additional risk (variance) that an indefinitely small additional investment into the equity would add to the market portfolio. A further conclusion of the model is that market betas alone suffice to explain the cross-section of expected equity returns. Although authors such as Black (1972) and Fama & MacBeth (1973) offer initial support for the model, subsequent studies have in general been able to reject the CAPM on data from 1963 to the present (Fama & French 1992). Concurrent with the attempts to support or reject the role of the market beta, substantial work has focussed on the impact of other firm-characteristics that could either add to the explanation of the cross-section of average returns as provided by the market betas or possibly even outplay the role of the market betas. In contradiction to the predictions of the CAPM many authors have provided empirical evidence that some easily observable firm characteristics indeed greatly add to the explanation of cross-sectional returns. Among these characteristics, the size of a firm (as measured by its market equity), cf. Banz (1981), and its book-to-market equity, cf. Stattman (1980) and Rosenberg, Reid & Lanstein (1985) are probably the most important ones. The role of size and book-to-market is strengthened by the findings of Fama & French (1992) who show that these two firm characteristics on their own capture cross-sectional variation in average stock returns associated with market beta, size, leverage, book-to-market equity and earnings-price for the period from 1963-1990.

The link between size, as measured by market equity (stock price times shares outstanding), and the cross-section of expected (average) equity returns is often referred to as the size effect. In empirical studies, the size effect is embodied in a negative mean slope coefficient obtained from monthly cross-sectional regressions of equity returns on size, thus implying that small firms perform on average better than large firms. The significance of the size effect is verified in numerous studies following the original paper by Banz (1981), for example by Fama & French (1992) and Knez & Ready (1997). However, Knez & Ready (1997) use least trimmed squares (LTS) regression techniques to show that, in contrast to the book-to-market effect, the size effect is driven by a few extreme observations and can therefore be reversed by sorting out less than one percent of these observations. In this study, the findings of Knez & Ready (1997) are extended by using an alternative statistical method to find out which firm-month observations most strongly contribute to the size anomaly.

The study investigates the robustness of the size effect for the time period from 1963 to 2007 and aims to shed further light on several properties of the negative association between size and equity returns. In Section 2, I introduce the applied econometric methods used in this study; i.e., ordinary least squares (OLS), least trimmed squares (LTS) (Rousseeuw & Van Driessen 2002) and a method I shall call ‘leverage least squares’ (LLS) (Davidson 2004). Following the ideas by Fama (1976) I will show in Section 3 that the slope coefficients from an OLS regression of equity

returns on firm characteristics can be interpreted as the returns of portfolios, which (a) consist of all equities used in the cross-sectional regressions and which (b) have a unit exposure towards the associated firm characteristic and a zero exposure towards all other firm characteristics in the regression. The average returns of these portfolios are therefore a clean measure of the excess compensation required by equity investors to be compensated for the risk associated with the firm characteristic. The ultimate goal of this study is to find out which firm-month observations most strongly contribute to the average slope coefficients and hence the average returns of the mimicking portfolios. Knez & Ready (1997) use the LTS method for this purpose. However, in contrast to the LLS method, the LTS method sorts out observations based on the magnitude of their regression residuals, potentially eliminating observations which diminish anomaly effects. As a result, I conclude that the LTS method is not ideally suited to identify the firm-month observations which most strongly drive characteristic effects. In comparison, the LLS method, which sorts out observations based on the difference between the full-sample slope coefficient and the slope coefficient obtained when excluding one or more observations, is better suited for this purpose. In section 4, I present empirical results obtained from the different regression techniques. First, I replicate the OLS and LTS regression results shown in Knez & Ready (1997) using the Fama-MacBeth procedure. In these tests, equity returns are regressed on size and book-to-market both individually and jointly. I supplement my findings by the results of the LLS regression method. The differences between the LTS and LLS methods are highlighted using scatter plots for a specific regression month, which most closely matches the average regression results. I also discuss the results of pooled univariate cross-sectional and time-series regressions of equity returns on size and book-to-market for the time period from 1963 to 2007. The pooled regressions allow me to investigate how many high leverage observations need to be sorted out to make the size and book-to-market effect vanish. The outliers are then plotted against time to identify the months that most strongly contribute to the size effect. The LLS method also allows me to test whether certain firms persistently contribute to the anomaly effects over time and to analyze if a firm's contribution to the size anomaly is related to other firm characteristics, such as liquidity, financial distress or inclusion in a broad stock market index. The following conclusions can be drawn from the empirical analysis.

The results obtained from the LLS method with 1% and 5% trimming proportions corroborate the results by Knez & Ready (1997) that the size anomaly is strongly driven by just a few small firms with highly above average returns. In fact, I show that it suffices to trim 0.14% of the observations with highest leverage on the size slope coefficient to make the size effect disappear. Plotting this small fraction of high leverage observations on the time axis, reveals that the time periods which most strongly contribute to the size effect are often towards the end or shortly after a recession period, which indicates that the size effect is strongly driven by the recovery of small firms after a period of recession. This implies, that portfolio managers that are able to anticipate the end of a recession period could possibly profit from a trading strategy that switches to small firms in these time periods, although the high transaction costs of small firms might be an obstacle here. I apply another statistic test according to Chan et al. (2003) to test the persistence of high leverage on the size effect over time. The results of the test imply that although only very few firms consistently contribute to the size-effect, there appears to be a small number of firms that actually shows strong persistence over subsequent month. Obviously, identifying these firms bears some potential to improve on size trading strategies. Finally, I run cross-sectional regressions of size-leverage on frequently used measures of firm's size, liquidity, financial distress and inclusion in a broad stock market index. The results of these regressions imply that there exists a negative relation between a firm's size and liquidity on the one hand and its

leverage on size on the other hand. Taking the results of this study into account, prospective empirical work may focus on the improvement of size trading strategies, which account for the temporal and cross-sectional characteristics of the observations driving the size effect. However, since it appears to be mostly small firms that drive the size effect, the considerations have to account for the high transaction costs and high price impact that are encountered when trading small firms. Thus, the question remains if a size trading strategy, which relies on a few small firms having extreme positive returns, is really feasible for real-world investors, or if these firms are in the end ‘un-investable’, due to prohibitively high transaction costs.